

BERGVLIET HIGH SCHOOL
MATHEMATICS

GRADE 12

MAY 2010

EXAMINER L STOREY

150 MARKS

3 hrs

INSTRUCTIONS

WRITE YOUR NAME AND YOUR TEACHERS NAME ON YOUR SCRIPT

Read the following instructions carefully before answering the questions.

1. There are 8 pages and 11 questions. Answer ALL questions.
2. Clearly show ALL calculations and diagrams you have used in answering.
3. A non programmable calculator may be used to answer the questions.
4. If necessary, answers should be rounded off to TWO decimal places.
5. Number the answers EXACTLY as the questions are numbered.
6. It is in your best interests to write legibly and to present the work neatly

QUESTION 1

Solve for x (correct to two decimal places if necessary)

1.1. $2x^2 - x - 5 = 0$ (3)

1.2. $2x(x-2) = x-9$ (3)

1.3. $\frac{x-2}{x-1} = \frac{2x-1}{x+7}$ (5)

1.4. $\frac{6x+1}{2x-3} \geq 1$ (6)

1.5. $6 - \sqrt{x+2} = x+2$ (5)

1.6. $\log_x \frac{1}{25} = -2$ (2)

1.7. $9^x = \frac{\sqrt{3}}{3}$ (3)

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QUESTION 2

Solve for x and y

$$x - 2y = 3$$

$$4x^2 - 5xy = 3 - 6y$$

[7]

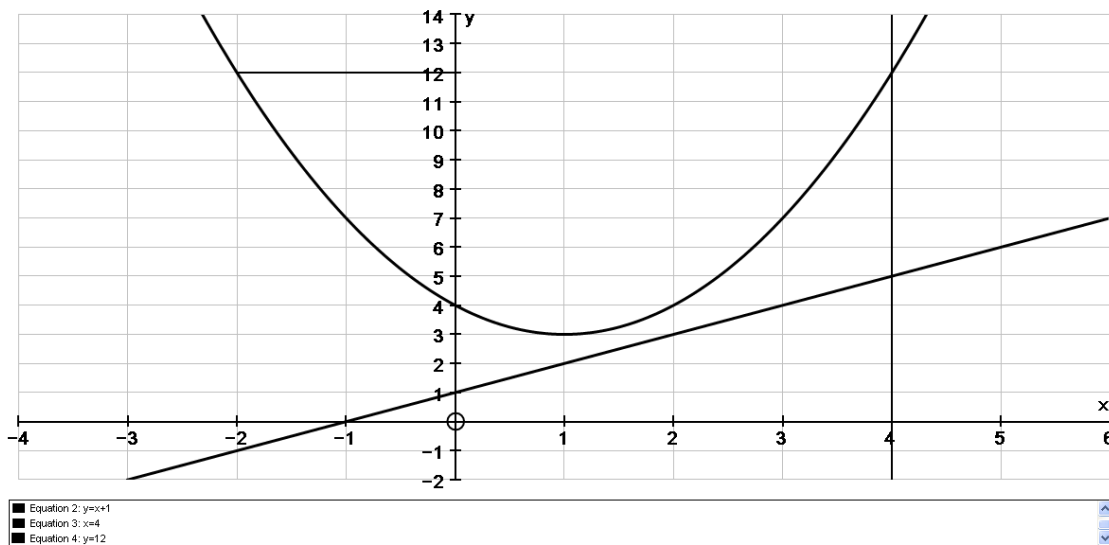
QUESTION 3

The graph below represents the functions

$$f(x) = x^2 - 2x + 4 \text{ and } g(x) = x + 1 \text{ where}$$

DM is perpendicular to the y axis

AB is perpendicular to the x axis and Q is the turning point of $f(x)$



3.1. Write $y = x^2 - 2x + 4$ in the form $y = (x - p)^2 + q$ and then give the coordinates of Q (4)

3.2. Calculate the co-ordinates of D if OM = 12 (4)

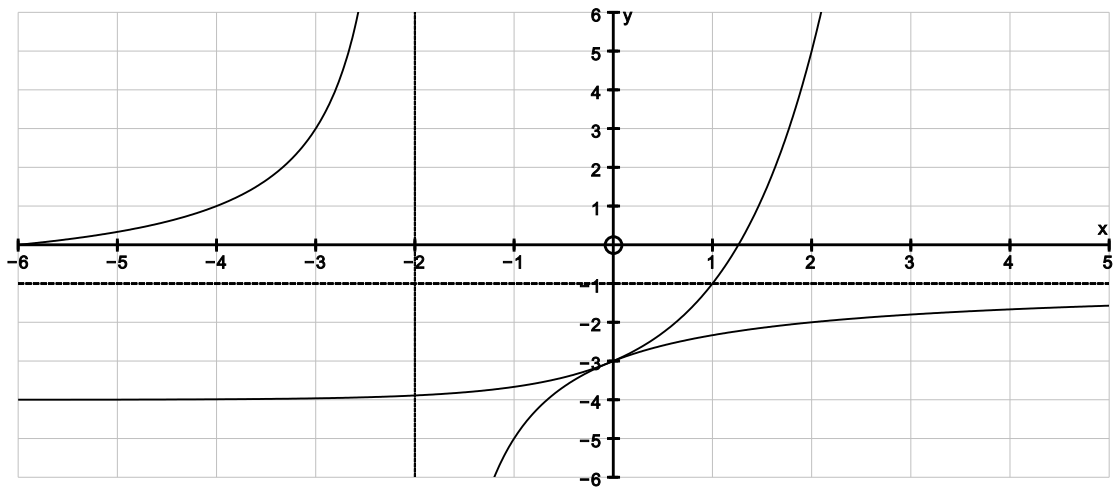
3.3. Write down the value of k (the y co-ordinate of A) (1)

3.4. Find the length of AB (2)

3.5. Calculate the minimum distance between $f(x)$ and $g(x)$ (5)

[16]

QUESTION 4



$$f(x) = b^x + c \quad \text{and} \quad g(x) = \frac{a}{x+p} + q$$

- 4.1. For which value of x is $f(x) = g(x)$? (1)
- 4.2. For which values of x is $f(x) > g(x)$? (2)
- 4.3. Write down the equation of the asymptote of f . (1)
- 4.4. Determine the equation of f . (2)
- 4.5. Write down the equations of the asymptotes of g . (2)
- 4.6. Determine the equation of g . (3)
- 4.7. What are the equations of the axes of symmetry for g ? (2)

[13]

QUESTION 5

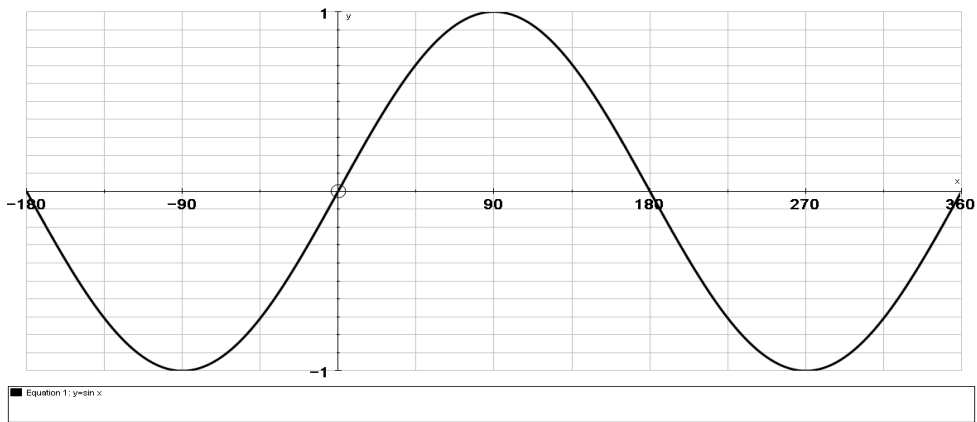
5.1. If $h(x) = 2x^2$

5.1.1. Determine the equation of the inverse of h . (2)

5.1.2. How can we restrict the domain of $h(x) = 2x^2$ to make the inverse a function? (2)

5.1.3. Sketch the graph of h and its inverse. (2)

5.2. Given $f(x) = \sin x$



5.2.1. What will the equation of the above graph become if it is reflected in the x axis and moved 35° to the right. (2)

5.2.2. How can the above graph be shifted so as to become the graph $y = \cos x$ (2)

[10]

QUESTION 6

6.1.

6.1.1 Which is the better investment A or B . All calculations must be shown

A An amount x invested at a simple interest rate of 12% pa for 8 years.

B An amount x invested at a compound interest rate of 10% pa for 8 years. (3)

6.1.2. What compound interest rate would yield the same amount as investment **A**? (2)

6.2. The annual depreciation of a certain car is 15% based on a reducing balance method. If the new price is R120 000 after how many years will the car be worth R40 000. (Correct to two decimal places) (3)

6.3. Joan invests R10 000 for 3 years at a nominal interest rate of 15% pa compounded quarterly. What was the effective rate per annum at which her money grew. (3)

[11]

QUESTION 7

Mr. Clark loans his daughter R100 000 in order to start her own business.

She agrees to pay him back R20 000 after 3 years.

R x after a further two years (ie 5 years from the time of the original loan) and R2 x 3 years after that. Mr. Clark charges her 10% compound interest pa compounded quarterly for the first 4 years and 14% pa compounded annually for the rest of the time. If the loan is fully paid after 8 years, calculate the value of x .

[6]

QUESTION 8

8.1. If $f(x) = \frac{3}{x}$ find the derivative using FIRST PRINCIPLES

(5)

8.2 Find the limit of the following if it exists

• $\lim_{x \rightarrow -1} \frac{x^2 - 4x - 5}{x + 1}$

(3)

8.3. Find $\frac{dy}{dx}$ in each of the following

8.3.1. $y = 3(2x-1)^2$

(3)

8.3.2. $y = \frac{5x^2 - 4x}{x}$

(2)

[13]

QUESTION 9

Given $f(x) = -x^3 + 10x^2 - 17x - 28$

9.1. Find the x and y intercepts.

(4)

9.2. Determine the turning point showing all calculations.

(2)

9.3. Find the point of inflection.

(1)

9.4. Draw the graph of $f(x)$

(3)

[10]

QUESTION 10

10.1. A stone is thrown vertically upwards and its height (in metres) above the ground is given by : $h(t) = 35 - 5t^2 + 30t$

10.1.1. Find the initial height of the stone above the ground. (1)

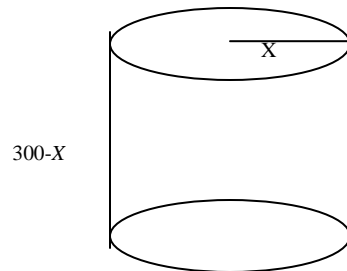
10.1.2. Find the initial speed with which it was thrown (2)

10.1.3. Find the maximum height above the ground that the stone reached (3)

10.1.4. How fast was the stone traveling when it reached a height of 60m above the ground on the way down? (4)

10.1.5. How fast was it traveling when it hit the ground? (3)

10.2.



The radius of the vase of a solid metal cylinder is x mm while its height is $(300-x)$ mm.

10.2.1. Show that the volume of the cylinder is given by

$$v = 300\pi x^2 - \pi x^3 \quad (2)$$

10.2.2. Find the value of x which will result in a maximum volume. (5)

10.2.3. What is the maximum volume? (2)

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QUESTION 11

To encourage spectator support at the World Cup soccer event, authorities consulted a local manufacturer to produce *vuvuzelas* (a plastic trumpet that makes a unique South African sound). The manufacturer agrees to produce two types Type X (a drinkazela which can be used to drink out of and blow) and Type Y (the usual kind). On a weekly basis, he can produce a maximum of 300 of type X and a maximum of 400 of Type Y. His machinery can cope with a maximum of 500 per week of X and Y combined. The number of type Y must be at least half of the number of type X.

- 11.1. Represent the above information as a system of inequalities. (4)
- 11.2. Draw on the graph paper provided the above inequalities and indicate the feasible region. (5)
- 11.3. If the profit on Type X is R3 and the profit on Type Y is R2
Write down a profit equation P in terms of X and Y. (1)
- 11.4. Use the graph to determine the number of each that must be produced each week to maximize the profit. (3)
- 11.5. What is the maximum weekly profit? (2)

[15]