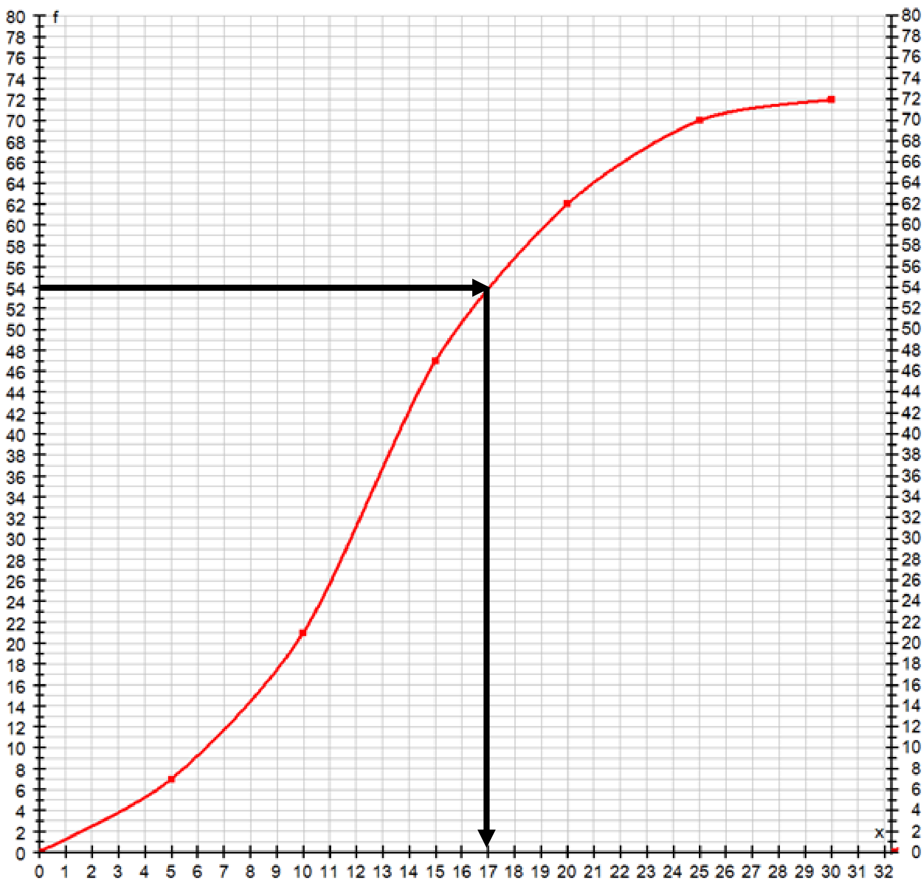
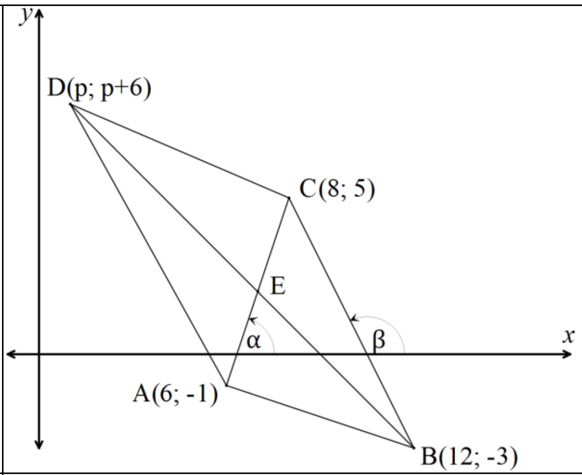
Memo – Bergvliet High School – G12 Mathematics P2 – September 2015 – C Smith

1.1.1)	Median	✓a	(1)
1.1.2)	C *(interquartile range is 28)*	✓a	(1)
1.1.3)	20	✓a	(1)
1.1.4)	C	✓a	(1)
1.1.5)	True The median is 54% and the median indicates the middle value in a set of data.	✓a (no reason = 0/2) ✓reason	(2)
1.1.6)	B	✓✓a	(2)
1.1.7)	Some students may have obtained a higher mark while others a lower mark leaving the other quartiles with the same result. Quartiles are not an indication of improvement.	✓ any valid reason	(1)

1.2.1)	72	✓✓a	(2)
1.2.2)	62 obtained 20 and 21 obtained 10. 62 – 21 = 41 students	✓✓a	(2)
1.2.3)		✓✓ arrow at 54 ✓ 17 indicated somewhere Give ½ for any other integral endpoint which has been labelled.	(3)
1.2.4)	6 to 10 or $5 < x \leq 10$ or $x \in (5;10]$	✓ lower bound ✓ upper bound	(2)
			[18]

	 		
2.1.1)	$E\left(\frac{8+6}{2};\frac{5+(-1)}{2}\right)$ $=E(7;2)$	✓7 ✓2	(2)

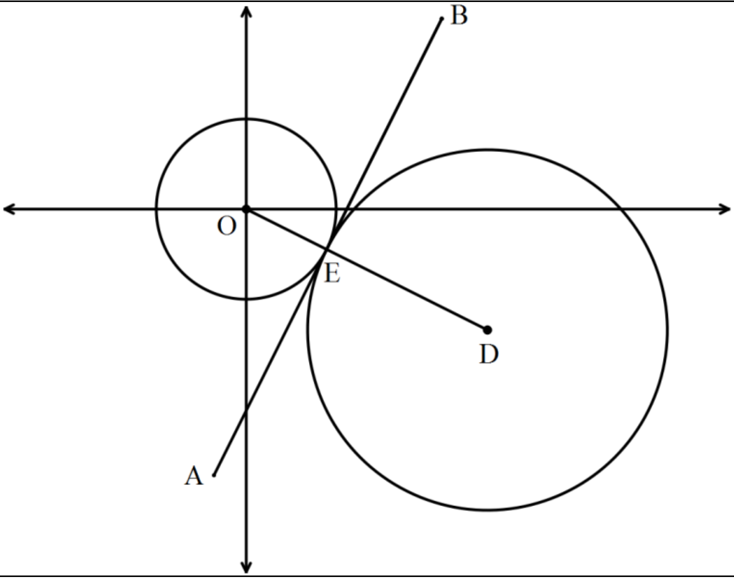
2.1.2)	$y-y_1=m(x-x_1)$ $y+3=\frac{2+3}{7-12}(x-12)$ $=-\frac{5}{5}(x-12)$ $=-x+12$ $y=-x+9$	✓sub ✓gradient ✓a	(3)

2.1.3)	Sub (p;p+6) $\therefore p+6=-p+9$ $\therefore 2p=3$ $\therefore p=\frac{3}{2}$ $\therefore D\left(\frac{3}{2};\frac{15}{2}\right)$ *or* $D(1,5;7,5)$	✓sub ✓✓ co-ordinates	(3)

2.1.4)	$AC^2=(8-6)^2+(5+1)^2$ $=40$ $AC=2\sqrt{10}$ $BC^2=(12-8)^2+(-3-5)^2$ $=80$ $BC=4\sqrt{5}$ (decimal values for info only AC = 6,32155532 BC = 8,94427191)	✓sub ✓a ✓sub ✓a	(4)

2.1.5)	$\tan\alpha=\frac{6}{2}=3$ $\therefore \alpha=71,56505118°=71,6°$ $\tan\beta=-2$ $\therefore \beta=180-26,6°=116,5650512=116,6°$	✓tan✓gradient ✓a ✓gradient ✓a	(5)

2.1.6)	$\hat{ACB} = 116,6° - 71,6°$ $= 45°$ $\therefore \text{Area } \Delta ABC = \frac{1}{2} AC.BC.\sin \hat{ACB}$ $= \frac{1}{2}.2\sqrt{10}.4\sqrt{5}.\sin 45°$ $= 20 \text{ cm}^2$	✓a ✓f ✓sub with ca from 2.1.4 & 2.1.5 ✓ca	(4)



2.2.1)	$x^2 + y^2 = r^2$ $E(2;-1)$ $2^2 + (-1)^2 = r^2 = 5$ $\therefore x^2 + y^2 = 5$	✓sub ✓a	(2)

2.2.2)	$r_{big} = 2r_{small}$ $\therefore r_{big} = 2\sqrt{5}$ $\therefore r^2{}_{big} = 20$ *{ca students value for radius of larger circle}* $(x-a)^2 + (y+3)^2 = 20$ sub E(2;-1) $\therefore (2-a)^2 + (-1+3)^2 = 20$ $\therefore 4 - 4a + a^2 + 4 = 20$ $\therefore a^2 - 4a - 12 = 0$ $\therefore (a-6)(a+2) = 0$ $\therefore a = 6 \text{ or } a = -2$	✓radius big circle ✓formula ✓sub ✓factorised	(4)

2.2.3)	$(x-6)^2 + (y+3) = 20$	✓*(x-6)* ✓*(y+3)* *ca values from above* ✓20	(3)

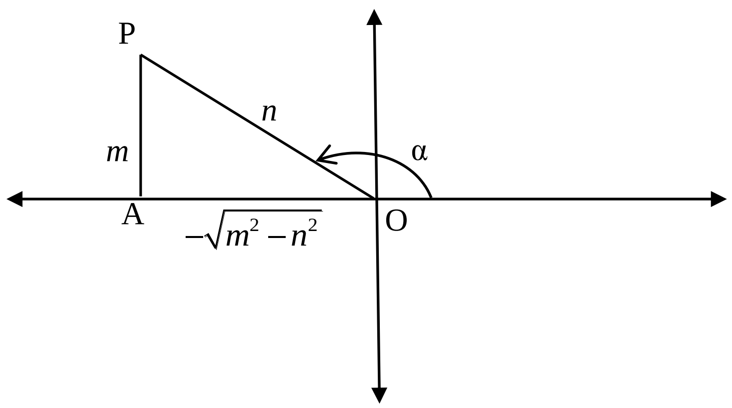
2.2.4)	$m_{radius\ small} = \frac{-1}{2}$ $\therefore m_{radius\ \tan} = 2$ $\therefore y+1 = 2(x-2)$ $\therefore y = 2x-5$	✓a ✓a ✓sub ✓a	(4)

2.3.1)	$x^2+y^2-4x+5y+k=0$ $\therefore x^2-4x+\mathbf{2}^2+y^2+5y+\left(\frac{\mathbf{5}}{\mathbf{2}}\right)^2 = -k+4+\frac{25}{4}$ $\therefore (x-2)^2+\left(y+\frac{5}{2}\right)^2 = -k+\frac{41}{4}$ $\therefore \text{Centre}\left(2;-\frac{5}{2}\right)$	✓method ✓method ✓x-coord ✓y-coord	(4)

2.3.2)	$-k+\frac{41}{4} = 3^2$ $\therefore k = \frac{5}{4}$	✓equation ✓radius = 3 ✓a	(3)
			[41]

3.1)	$\cos(A+B) = \cos A.\cos B - \sin A.\sin B$ $\therefore \cos(A+A) = \cos A \cos A - \sin A \text{Sin } A$ $\therefore \cos(2A) = \cos^2 A - \sin^2 A$ $\therefore \cos(2A) = \cos^2 A - (1-\cos^2 A)$ $\therefore \cos(2A) = 2\cos^2 A - 1$	✓sub 'A' ✓a ✓identity {Given}	(3)

3.2.1)			



	$\sin\alpha = \frac{m}{n}$ $\therefore$ quad = 2nd $\therefore 3rd\ \text{side} = -\sqrt{m^2-n^2}$ (Pythagoras' theorem) $\therefore \cos\alpha = \frac{-\sqrt{m^2-n^2}}{m}$	✓a ✓quad ✓3rd side ✓a	(4)

3.2.2)	$\cos(\alpha+60°)=\cos\alpha.\cos60°-\sin\alpha\sin60°$ $=\cos\alpha.\frac{1}{2}-\sin\alpha.\frac{\sqrt{3}}{2}$ $=\frac{1}{2}\left(-\frac{\sqrt{m^2-n^2}}{m}\right)-\frac{\sqrt{3}}{2}\left(\frac{n}{m}\right)$ $=\frac{-\sqrt{m^2-n^2}-\sqrt{3}n}{2m}$ $=\frac{-\sqrt{m^2-n^2}}{2m}-\frac{\sqrt{3}n}{2m}$	✓expansion ✓a ✓sub ✓a (either)	(4)

3.3)	$\cos(90°+x)\tan(540°+x)\cos(x-180°)+\sin(-90°)$ $=-\sin x.\tan x.-\cos x+(-1)$ $=\sin x.\frac{\sin x}{\cos x}.\cos x-1$ $=\sin^2 x-1$ $=-\cos^2 x$	✓-sinx ✓tanx ✓-cosx ✓-1 ✓a ✓a	(6)

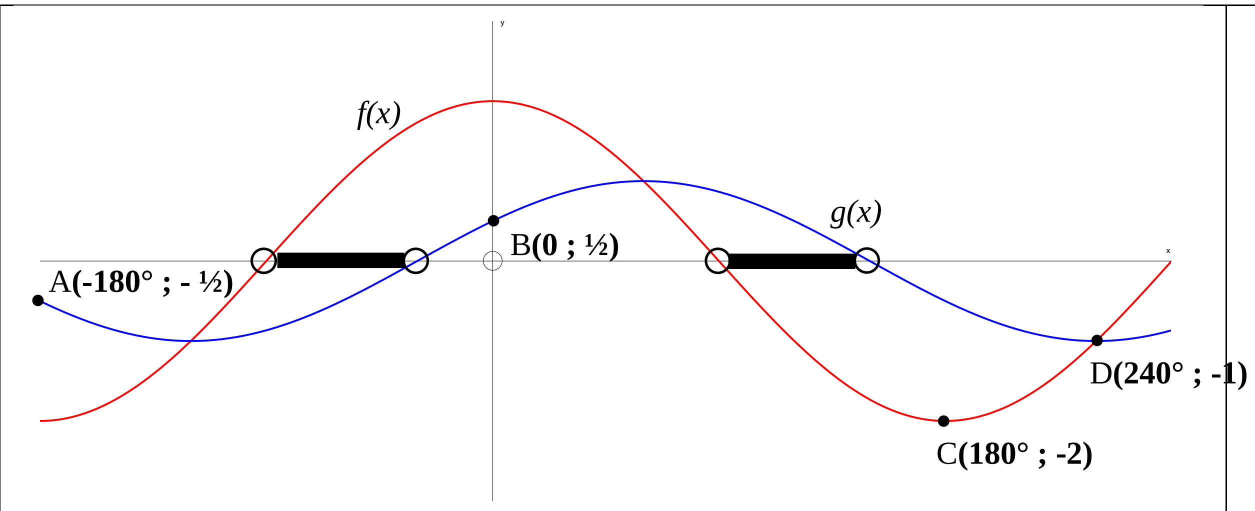
3.4.1)	LHS = $(\sin x+\cos x)^2$ $=\sin^2 x+2\sin x\cos x+\cos^2 x$ $=2\sin x\cos x+1$ $=\sin 2x+1$ $=$ RHS	✓ product ✓a	(2)

3.4.2)	$3\sin5x+3\cos5x \quad =3\left(\sqrt{(\sin5x+\cos5x)^2}\right)$ $=3\left(\sqrt{\sin10x+1}\right)$ now maximum $\sin10x \quad =1$ $\therefore$ max $\sin10x+1 \quad =2$ $\therefore$ max $\sqrt{\sin10x+1} \quad =\sqrt{2}$ $\therefore$ max $3\left(\sqrt{\sin10x+1}\right) \quad =3\sqrt{2}$	✓a ✓a ✓a ✓a	(4)
			[23]

4.1)	$\sin(x+30°)=\sin x\cos 30°+\cos x\sin 30°$ $=\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x$	✓a ✓a	(2)

4.2)	$2\cos x=\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x$ $\therefore\ \frac{3}{2}\cos x=\frac{\sqrt{3}}{2}\sin x$ $\therefore\ \frac{3}{2}\div\frac{\sqrt{3}}{2}=\frac{\sin x}{\cos x}=\tan x$ $\therefore\ \tan x=\frac{3}{\sqrt{3}}=\sqrt{3}$ $\therefore$ ref $\angle$ $=60°$ $\therefore\ x=60°+k180°\quad x\in\mathbb{Z}$ $\therefore\ x=-120°\ or\ x=60°\ or\ x=240°$	✓sub ✓simplify ✓tan ✓a ✓ref angle ✓a (all or nothing)	(6)

4.3)

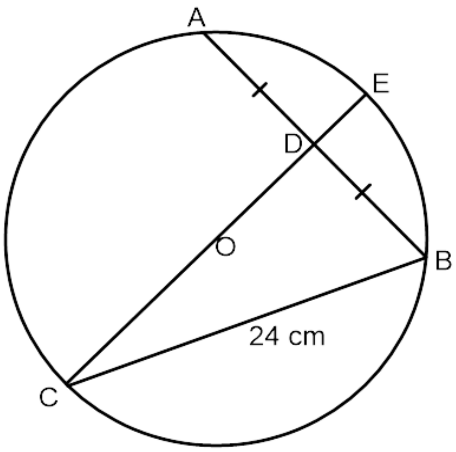


4.3.1)	labels	✓ both correct	(1)

4.3.2)	A(-180 ; -½)	✓ y-coord	
	B(0 ; ½)	✓ y-coord	
	C(180 ; -2)	✓ y-coord ✓ x-coords A, B & C	
	D(240° ; -1)	✓ x-coord ✓ y-coord	(6)

4.4)	See sketch	✓✓ each interval ✓ all end points	(3)
			[18]

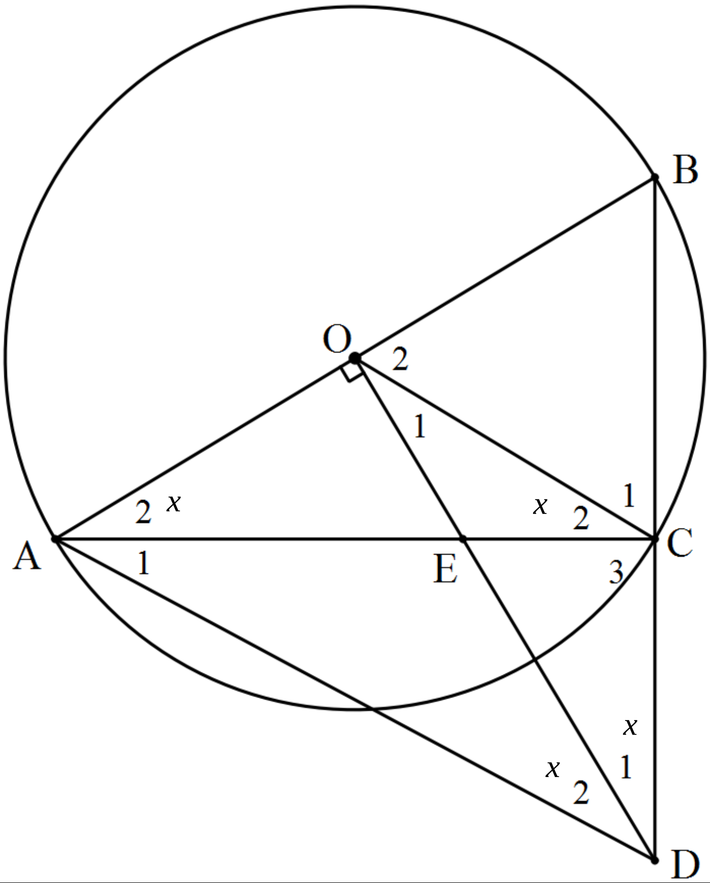
5.1)	A line drawn from the centre of a circle to the midpoint of the chord is **perpendicular** to the chord.	✓	(1)

5.2)			
5.2.1)	DB = 9cm (given) $\hat{CDB} = 90°$ (line from centre to mid pt chord) $\therefore BC^2 = CD^2 + DB^2$ (pyth theorem) $\therefore CD^2 = 24^2 - 9^2 = 495$ $\therefore CD = \sqrt{495} = 3\sqrt{55}\ cm$	✓a ✓a+r ✓a+r ✓a	(4)

5.2.2)	$\frac{CD}{DE} = 3$ $\therefore DE = \frac{CD}{3} = \frac{3\sqrt{55}}{3} = \sqrt{55}$	✓subst ✓a	(2)

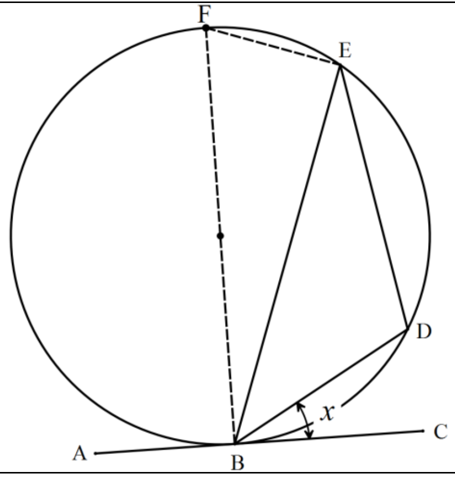
5.3)	Prove: 1. Opposite angles are supplementary 2. Exterior angle equal to the interior opposite angle 3. Angles subtended by the same side are equal	✓ ✓ ✓	(3)

5.4)

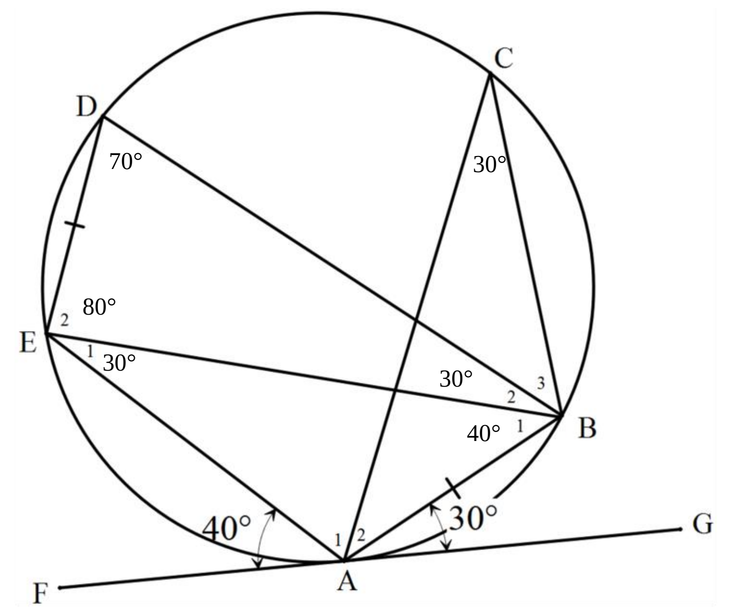


5.4.1)	$A\hat{O}D = 90°$ (given) $A\hat{C}B = 90°$ (angle in semi circle) $\therefore A\hat{O}D = A\hat{C}B$ (but these are subtended by AD) $\therefore$ AOCD is a cyclic quad (converse ∠'s in same segment)	✓a ✓a+r ✓a ✓r	(4)

5.4.2)	Given $\hat{C}_2 = x$ $\therefore \hat{D}_2 = x$ (∠s in same segment) $\therefore \hat{A}_2 = x$ (equal angles opp equal sides; radii) or (isos Δ; radii) $\therefore \hat{D}_1 = x$ (∠s in same segment)	✓a✓r ✓a✓r ✓a✓r	(6)
			[20]

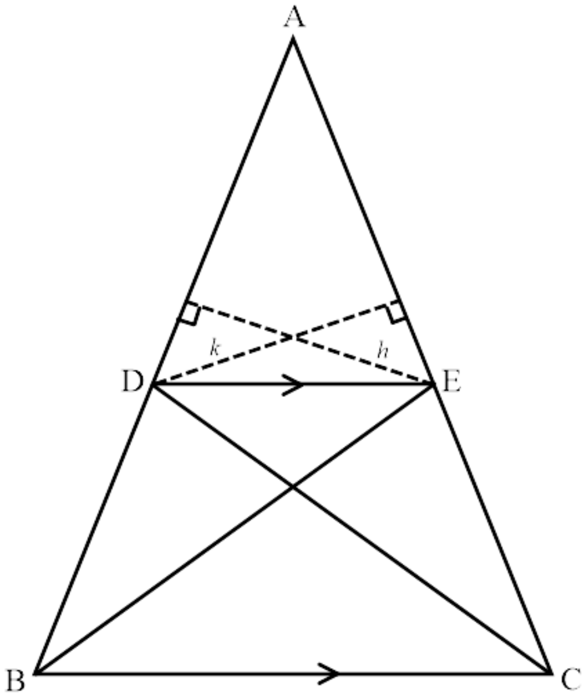
6.1)			
		Construction: Draw diameter BF and join F to E	
		Proof	
	6.1.1)	$\hat{BEF} = 90°$ **(angle in semi circle)** ✓	(1)
		Let $\hat{CBD} = x$	
	6.1.2)	$\hat{CBF} = \mathbf{90°}$ ✓ **(tan ⊥ radius) or (tan ⊥ diameter)**✓	(2)
		$\therefore \hat{FBD} = 90° - x$	
	6.1.3)	$\therefore \hat{FED} = 90° + x$ **(opp ∠s cyclic quad)**✓	(1)
	6.1.4)	$\therefore \hat{BED} = (90° + x) - 90° = \boldsymbol{x}$✓	(1)
		$\therefore \hat{CBD} = \hat{BED}$	

6.2)



6.2.1)	$\hat{E}_1 = 30°$ (tan chord theorem)	✓a ✓r	(2)
6.2.2)	$\hat{C} = 30°$ (tan chord theorem)	✓a+r	(1)
6.2.3)	$\hat{B}_2 = 30°$ (∠s subtended by equal chords)	✓a ✓r	(2)

6.2.4)	$\hat{B}_1 = 40°$ (tan chord theorem) $\therefore D\hat{B}A = 40° + 30° = 70°$ $\therefore D\hat{E}A = \hat{E}_1 + \hat{E}_2 = 110°$ (opp ∠s cyclic quad) $\hat{E}_1 = 30°$ (proved 6.2.1) $\therefore \hat{E}_2 = 80°$	✓a+r ✓a✓r ✓a	(4)
			[14]

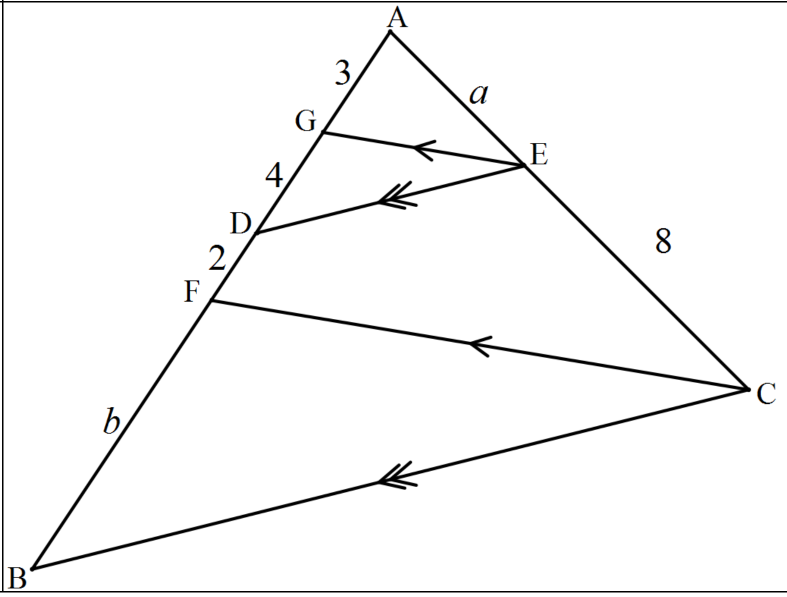
7.1)			

Construction: In ΔADE draw ⊥ height k from D to AE and ⊥ height h from E to AD.

Proof:

7.1.1)	$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}.AD.h}{\frac{1}{2}.BD.h} =$	$\frac{AD}{BD}$ ✓	(1)
7.1.2)	$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} =$ $\frac{\frac{1}{2}.AE.k}{\frac{1}{2}.EC.k}$ ✓	$= \frac{AE}{EC}$	(1)
7.1.3)	But area ΔDBE = area ΔDEC ✓	*(Same base and height/between same parallel lines)*	(1)
7.1.4)	$\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} =$	$\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ ✓	(1)
	$\therefore \frac{AD}{DB} = \frac{AE}{EC}$		

7.2)



7.2.1)	$\frac{AE}{EC}=\frac{AG}{GF}$ (prop int theorem ; GE| |FC) or (line | | to one side of Δ) $\therefore \frac{a}{8}=\frac{3}{6}$ $\therefore a=4$	✓a+r ✓sub ✓a	(3)
7.2.2)	$\frac{AD}{DB}=\frac{AE}{EC}$ (prop int theorem ; DE| |BC) or (line | | to one side of Δ) $\therefore \frac{7}{2+b}=\frac{4}{8}$ $\therefore b=12$	✓a+r ✓sub ✓a	(3)
7.2.3)	$\frac{\text{area } \Delta BFC}{\text{area } \Delta ABC}=\frac{BF}{AB}$ (common height) $=\frac{12}{21}=\frac{4}{7}$	✓a✓r ✓a	(3)
7.2.4)	$\frac{\text{area } \Delta AFC}{\text{area } \Delta ADE}=\frac{\frac{1}{2}AF.AC.\sin\hat{A}}{\frac{1}{2}AD.AE.\sin\hat{A}}$ $=\frac{AF.AC}{AD.AE}=\frac{9\times 12}{7\times 4}$ $=\frac{27}{7}$	✓a ✓sub {ca - use values from above} ✓a	(3)
			[16]