BERGVLIET HIGH SCHOOL

MATHEMATICS PAPER 2

GRADE 12   150 MARKS

SEPTEMBER 2010   3 HOURS

EXAMINER : L STOREY

INSTRUCTIONS:

Read the following instructions carefully before answering the questions.

1.   This paper consists a cover sheet, formula sheet 9 pages and two diagram sheets.
        Answer ALL the questions.

2.   Clearly show ALL calculations, diagrams, graphs etc. you have used in determining
        your answer.

3.   An approved scientific calculator (non-programmable and/or non-graphical) may be
        used, unless other instructions are given.

4.   If necessary, answers should be rounded off to TWO decimal places

5.   Number your answers **exactly** as the questions are numbered.

6.   Diagrams are NOT necessarily drawn to scale.

7.   It is in your own interest to write legibly and to present the work neatly.

8.   A diagram sheet has been provided for question 4.
        Hand this in with your answers.

9.   A formula sheet has been provided .

QUESTION ONE

1.1.   Refer to the figure below

A(-4;-1) , R(2;3) , and C(6;-3) are three points in the Cartesian plane.
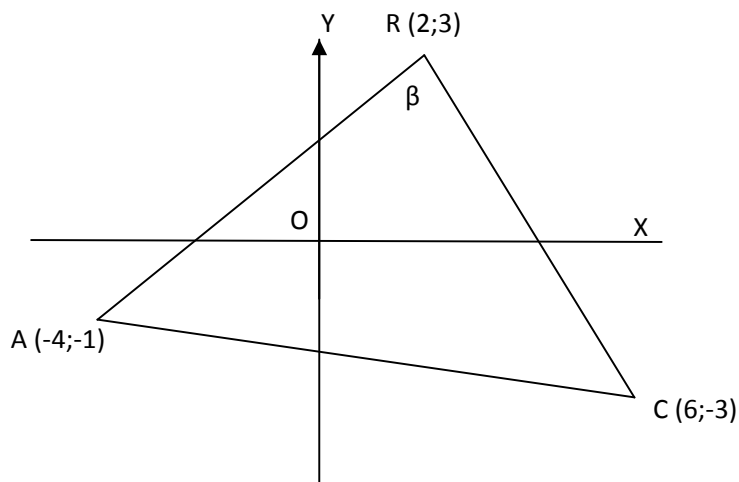
1.1.1. Calculate the coordinates of S, the midpoint of AC   (2)
1.1.2. Determine the equation of the line RC.   (3)
1.1.3. Determine the size of the angle β   (3)
1.1.4. Show that $\triangle$ ARC is an isosceles right angled triangle.   (6)

1.2.   In the diagram below, the line AB is defined by the equation $y = 2x + 5$ and the line CD is defined by the equation $y = -x + 2$.
The two lines intersect at K. The line EF is parallel to the *y*- axis, cutting CD and the *x* axis at H. EF intersects AB at G.
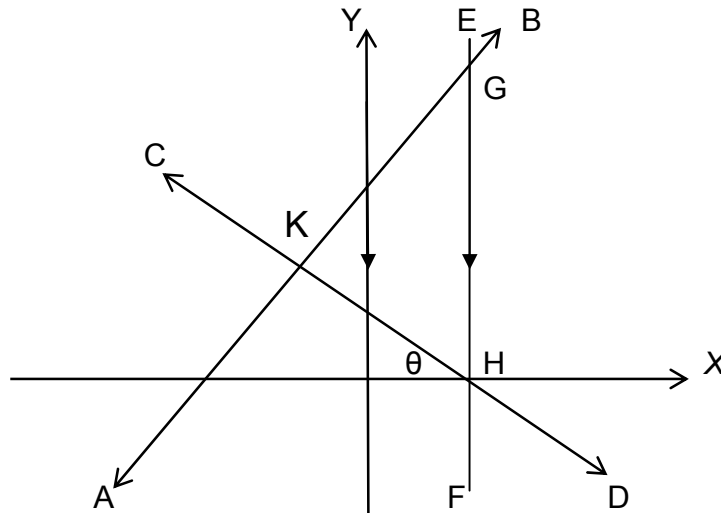
1.2.1   Write down the co-ordinates of H and G.   (3)

1.2.2   If AB and CD intersect at K, show that the co-ordinates of K are (-1;3)
                                                                                   (3)

1.2.3. Calculate the distance from K to G, leaving your answer in simplified surd

        form.   (2)

1.2.4. Find the size of θ.   (3)

                                                                                  [25]

QUESTION TWO

A circle with centre M is defined by the following equation:

$$x^2 + y^2 = 2y - 6x - 5$$

2.1   Find the radius of the circle AND the co-ordinates of the centre M.

(5)

2.2   Make a neat rough sketch of this circle, showing the co-ordinates

of M, as well as intercepts with the axes.   (5)

2.3   The line $x = -4$ intersects the circle at B in the third quadrant.

Find the equation of the tangent to the circle at B.

(7)

[17]

QUESTION THREE

The point A(4 ; − 3) undergoes the following transformations:

3.1.   A$(x ; y) \rightarrow$ A$'(x+3 ; y + 2)$

Name the type of transformation and give the co-ordinates of A'   (3)

3.2   If A is rotated 180° about the origin in a clockwise direction write

down the co-ordinates of the new point.   (2)

3.3   If A was rotated 90° write down the rule used in the translation.   (2)

[7]

QUESTION FOUR

4.1.   ANSWER THIS QUESTION ON THE DIAGRAM SHEETS

Given: O(0;0) ; P($-$3;1) and Q(1;2). Two transformations are given below

4.1.1. $(x ; y) \rightarrow (-y ; x)$ (6)

4.1.2. $(x ; y) \rightarrow (2x ; 3y)$ (6)

♦ On DIAGRAM SHEET 1 and DIAGRAM SHEET 2 provided, draw the
   image of ΔPQO according to each of the rules given above. Label each of
   your images including the new co-ordinates.

♦ On the diagram sheet identify the TYPE of transformation.

♦ On the diagram sheet give the ratio of $\dfrac{Area\, of\ \Delta PQO}{Area\, of\ \Delta P'Q'O'}$

4.2 If a point B (-3;1) is rotated anti-clockwise about the origin through an angle of

    150°, find the co-ordinates of the new point B'.   (3)

                                                                            [15]

QUESTION FIVE

5.1   Given: $\sin\theta = \dfrac{2k}{k^2+1}$,   *where* $0^0 < \theta < 90^0$

Make use of a sketch to find (in terms of *k*):

5.1.1 $\tan\theta$ (3)

5.1.2. $\sin(90^0 + \theta)$ (2)

5.2   Without the use of a calculator, simplify each of the following fully.

All steps must be shown. If only the answer is given, no marks

will be allocated.

5.2.1 $\dfrac{\cos^2(90^0 - x).\sin(90^0 + x)}{\cos(-x).\sin(-x).\cos(180^0 - x)}$ (7)

5.2.2 $\cos 79^0.\cos 34^0 + \sin 79^0.\sin 34^0$ (3)

5.2.3 $\cos(x - 30^0) + \sin(x - 60^0)$ (4)

5.3   Given : $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$

5.3.1   Prove the identity. (6)

5.3.2   For which value(s) of *x*, $0^0 \le x \le 180^0$ will the above identity be undefined.

(3)

5.3.3   Solve for *x*

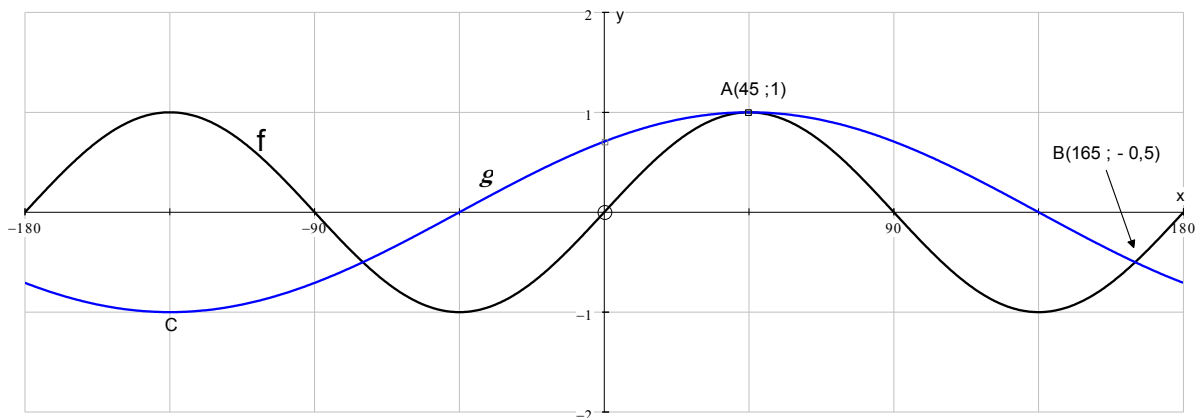$\tan(x - 30^0) = 5$ $x \in \left[0^0;360^0\right]$ (3)

5.3.4.   Find the general solution :

$8\cos^2 x - 2\sin x - 5 = 0$ (9)

[40]

QUESTION SIX

The graphs of $f(x) = \sin ax$ and $g(x) = \cos(x + b)$ for the interval $-180^0 \le x \le 180^0$ are sketched. Point $B(165^0; -0{,}5)$ is a point of intersection of $f(x)$ and $g(x)$.

Answer the following questions:

6.1   Find the values of *a* and *b*.   (3)

6.2   Write down the period of *g*.   (1)

6.3   For which values of *x* will:

      6.3.1 $g(x) < f(x)$ if $0^0 \le x \le 180^0$.   (2)

      6.3.2 $f(x) - g(x) = 2$ if $-180^0 \le x \le 180^0$ (2)

6.4   Determine the new equation of *f* if the *graph* is shifted 30^0 to the left.   (2)

6.5   Determine the new equation of *f* if the *graph* is shifted two units up and is then reflected in the x axis.   (2)

                                                                [12]

QUESTION SEVEN
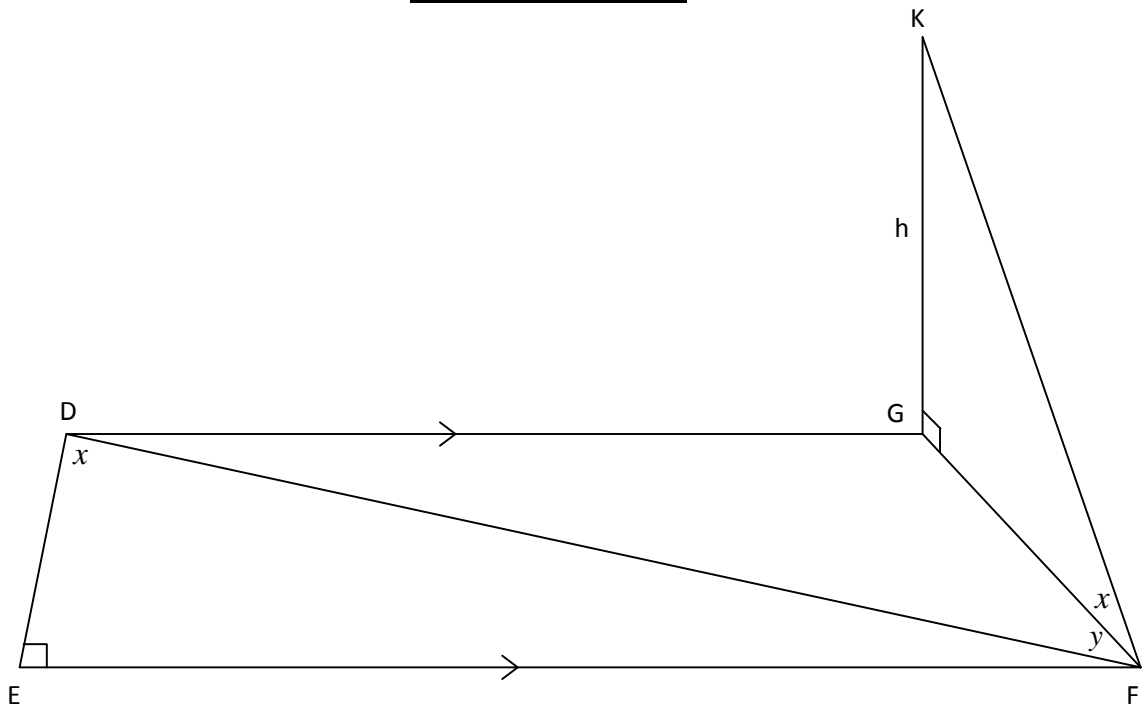
The accompanying diagram shows the boundaries of a sports field DEFG.

DG // EF and DE ⊥ EF. KG is a vertical pylon for a floodlight.

The angle of elevation of K from F is x. $E\hat{D}F = x$, $D\hat{F}G = y$ and $KG = h$ metres

7.1   Express $D\hat{G}F$ in terms of x and y.   (3)

7.2   Prove that $DF = \dfrac{h\cos(y-x)}{\sin x}$ (6)

7.3.   Find the length of DF if h= 20m , *x*=30° and *y*= 60 °   (2)

[11]

QUESTION EIGHT

David is bored in his English class, and so decides to keep record of the number of minutes he actually works in a 50 minute lesson. The results for ten lessons are given below:

22   24   26   27   27   31   33   38   42   50

8.1   Give the mode of the data.   (1)

8.2   Calculate the mean of the data.   (2)

8.3   Determine what percentage of the data lies within one standard deviation of the
       mean. Show ALL your calculations.

                                                                                                     (5)

8.4   Construct a box-and-whisker diagram from the data, clearly indicating
       the five-point summary of the given data.   (5)

8.5   Is this data "skewed"? If so, is the data positively skewed or negatively skewed?
       Give a reason for your answer, and explain what this means concerning the
       spread of the data.   (3)

                                                                                                    [16]

QUESTION NINE

On a reality show on DSTV, a group of 80 contestants was asked to estimate how long a piece of music had been played. Their estimates to the nearest second are given in the accompanying ogive:
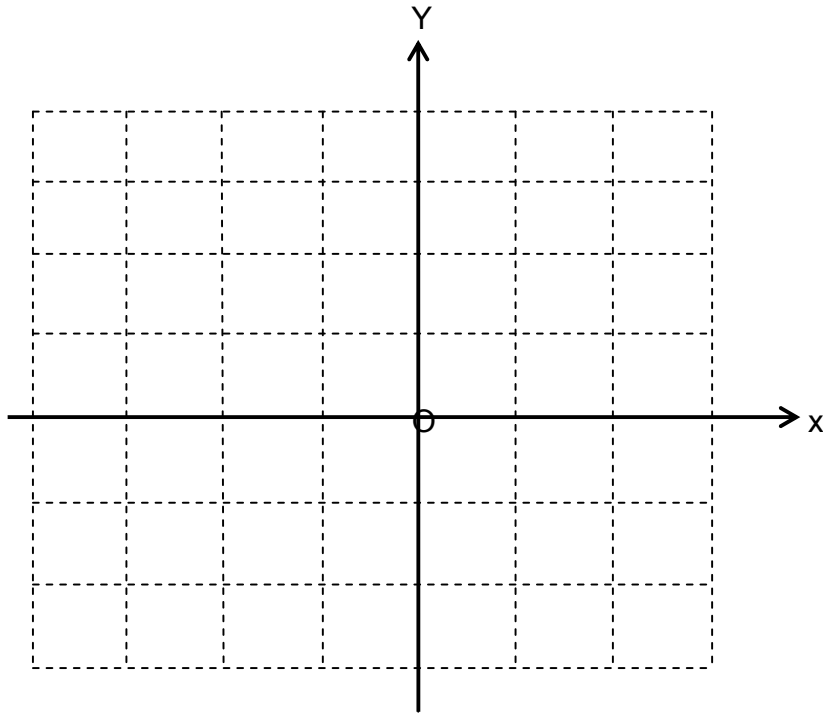
Cumulative Frequency

Estimated time (seconds)
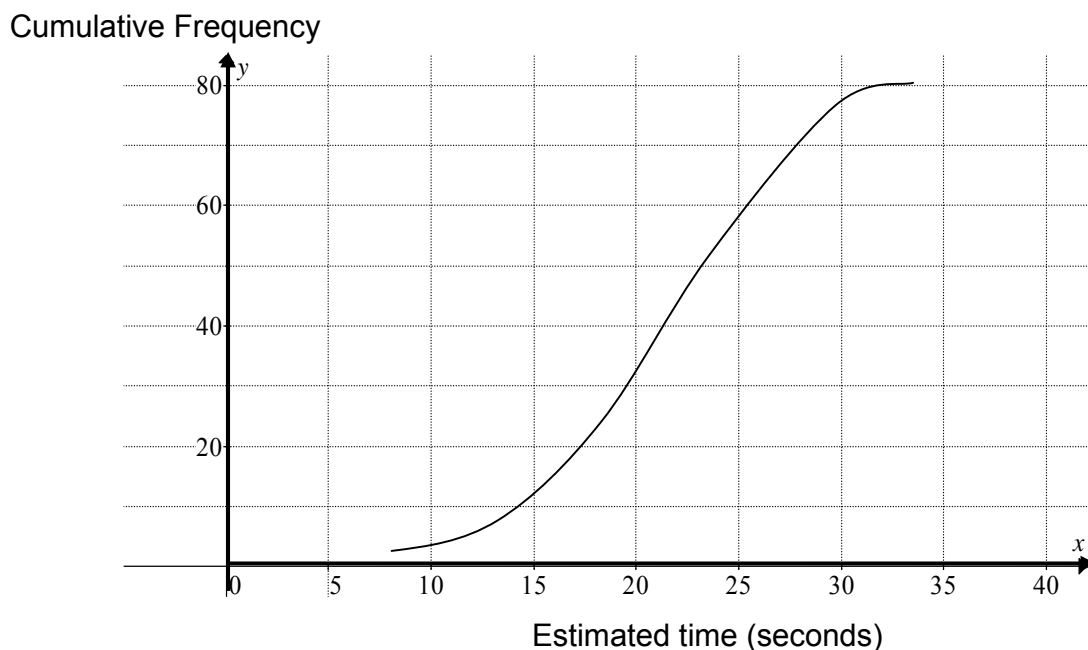
Use the graph to determine:

9.1   The median time estimated. (1)

9.2   The inter-quartile range of the time estimated. (3)

9.3   The number of contestants who over-estimated the time, which was actually 17 seconds. (2)

9.4   The value of the estimated time of the 70th percentile. (1)
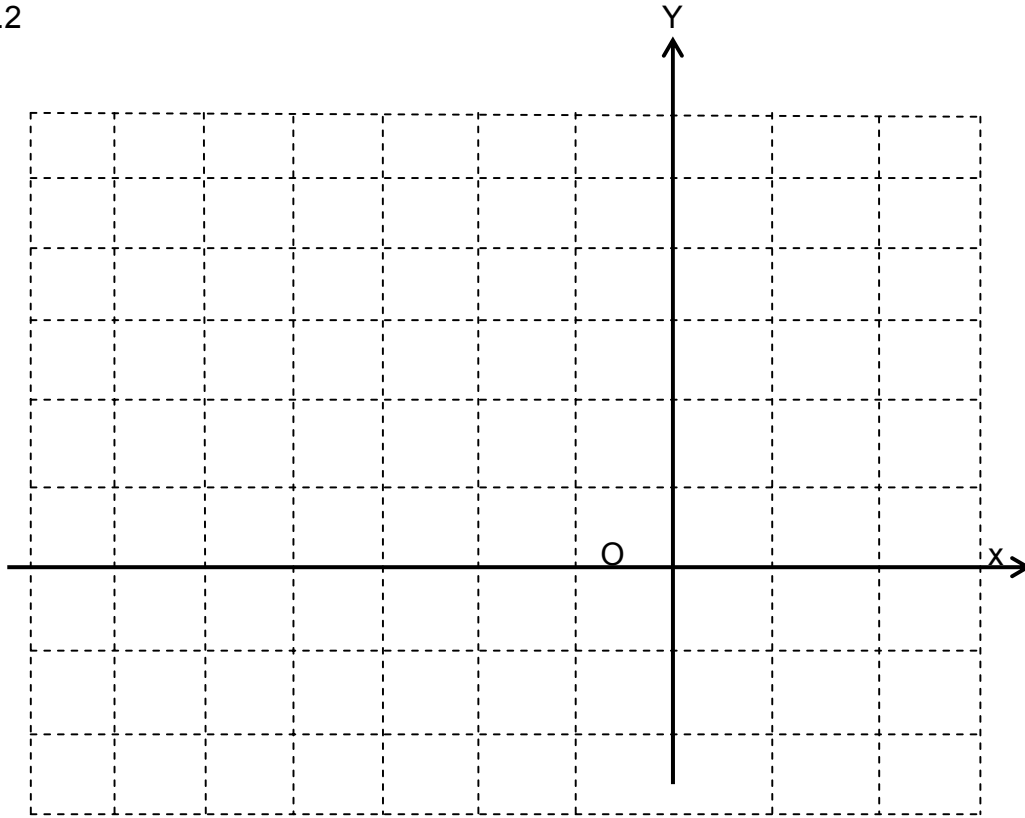
[7]

DIAGRAM SHEET 1

4.1.1

TYPE OF TRANSFORMATION: ....

RATIO; ....

[6]

DIAGRAM SHEET 2

4.1.2

TYPE OF TRANSFORMATION: ....

RATIO; ....

[6]